

Theory of gyroscopic effects for rotating objects

Ryspek Usubamatov

Kyrgyz State Technical University, Kyrgyzstan

More than two-century gyroscopic effects were presented analytically by the simplified models that did not match with practical results. The mathematician L. Euler described only one component of gyroscopic effects that is the change in the angular momentum. Other outstanding and ordinary scientists represented only some assumptions for gyroscopic properties. This is an unusual phenomenon in the physics of classical mechanics that can solve more complex problems than to compute forces acting on the simple spinning disc and its motions. The physics of the gyroscopic effects are more complex in mathematical models than represented in known theories. This problem solved by a new method based on the action of the system of inertial torques acting on the spinning objects that are produced by rotating mass. The action of nine interrelated inertial torques on spinning objects around three axes manifests all gyroscopic effects. Inertial torques are generated by the centrifugal, common inertial, Coriolis forces of the rotating mass, as well as the change in the angular momentum. This torques represents the fundamental principles of the gyroscope theory. Gyroscopic effects are described by mathematical models of the inertial torques and explained their physics based on the potential and kinetic energy conservation law. Mathematical models for the gyroscopic effects are validated by practical tests. The new solution is represented as the breakthrough gyroscope theory. All problems of mechanical gyroscopes are resolved and closed the unresolved problem in classical mechanics.

Keywords. Inertial torques, gyroscope theory, mathematical model.

1. Introduction: Numerous machines and mechanisms with rotating components manifest gyroscopic effects, i.e., the action of the inertial forces and mo-

tions of rotating objects. Famous and ordinary

Scientists and researchers studied the gyroscope problems and derived some mathematical

Foundations for the inertial forces of rotating objects. The deep investigations and applied the theory of dynamics of the rotating mechanism emerged in the twentieth century. The known publications demonstrate that forces and motions of rotating objects represent a specific and important area in engineering [1-4]. The dynamics of rotating objects and mathematical solutions for gyroscopic effects represented in textbooks of classical mechanics. The known textbooks of classical mechanics and publications interpret the physics of gyroscopic effects only by Euler's principle of the angular momentum nut practice demonstrates the action of additional forces [5 – 9]. The unsolved problems of gyroscopic effects studied by researchers [10 – 13]. The new research of the gyroscopic effects has formulated mathematical models of the action of the interrelated inertial torques generated by the spinning masses of the disc [13 -16]. The analytical solutions for the gyroscope motions and the action of the system of inertial torques are well-matched with practices.

2. Methodology: New studies into the physical principles of gyroscopic motions have presented mathematical models for the resistance and precession torques, whose equations are shown in Table 1. The gyroscopic resistance torque is generated by the action of the centrifugal and Coriolis forces of the gyroscope's mass-elements. The action of the common inertial forces of mass- elements and the change in the angular momentum of the spinning disc, produces the precession torque. These resistance and precession torques act at the same time and are interrelated and strictly perpendicular to each other around their axes.

Type of torque generated by	Equation, (N.m)
Centrifugal forces, $J\omega_i^2$	$T_c = T_m = \frac{2\pi^2}{9} J\omega\omega_i$
Inertial forces, T_m	
Coriolis forces, T_{cc}	$T_p = (8/9)J\omega\omega_i$
Change in angular momentum, T_{am}	$T_{am} = J\omega\omega_i$
Resistance torque $T_c = T_{cc} + T_{cc}$	$T_r = \left(\frac{2\pi^2 + 8}{9}\right)J\omega\omega_i$
Precession torque $T_p = T_m + T_{am}$	$T_p = \left(\frac{2\pi^2 + 9}{9}\right)J\omega\omega_i$

Table 1 represents the equations of acting torques, which contain the following symbols: J is the mass moment of inertia of the disc around own axis; ω_i is the angular velocity of the precession of a spinning disc around axis i and ω is its angular velocity. The inertial torques originated along each axis express the kinetic energies of the spinning disc. The sum of these inertial torques of one axis in the absolute value is equal to the sum of the inertial torques of another axis. This statement represents the principle of the conservation of mechanical energy. The equality of the kinetic energies of the inertial torques is expressed by the equality of the inertial torques acting around their axes. For the spinning disc with the inclined axis on the angle γ and with the load torque T acting around axes ox and oy (Fig. 1), this equality is expressed by the ratio of the angular velocities ω_y and ω_x of the gyroscope around axes of rotation.

$$\omega_y = \left[\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma} \right] \omega_x$$

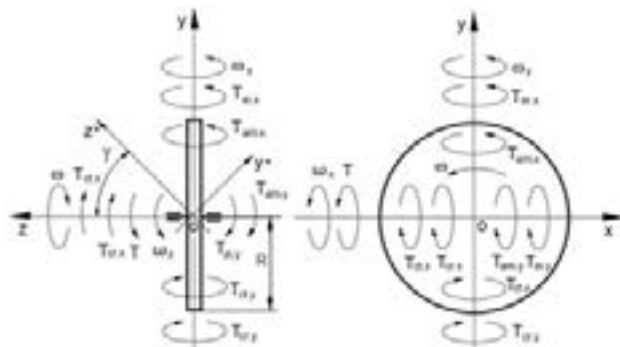


Figure 1. External and inertial torques acting around axes on the spinning disc

The equations of the load and internal torques acting on the spinning disc are used to formulate a mathematical model for the motions around two axes of a gyroscope suspended from the flexible cord. This example was the most unsolvable problem in the gyroscope theory. Figure 2 demonstrates the action of the load and inertial torques on the running gyroscope assembled with the ability to freely rotate around axes ox and oy on the cord.

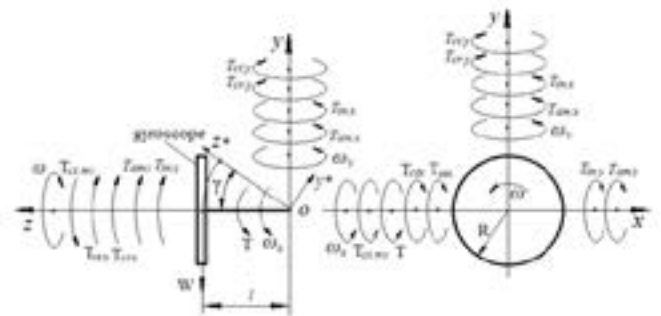


Figure 2. The torques and motions acting on the gyroscope with the spinning disc of the radius R suspended from the flexible cord.

The action of load torque T around axis ox is represented by the equation $T = Wgl \cos\gamma$,

$$J_x \frac{d\omega_x}{dt} = Wgl \cos\gamma - \left(\frac{2\pi^2 + 8}{9}\right)J\omega\omega_x - J\omega\omega_x \tag{2}$$

$$J_y \frac{d\omega_y}{dt} = \left(\frac{2\pi^2 + 9}{9}\right)J\omega\omega_x \cos\gamma - \frac{8}{9}J\omega\omega_x \cos\gamma \tag{3}$$

where W is the mass of the gyroscope, g is the gravity acceleration, γ is the angle of the gyroscope's axle inclination, l is the distance between the center mass of a gyroscope and support of a cord.

The mathematical model for the motions of the gyroscope suspended from a flexible cord around axes ox and oy is represented by Euler's differential equations as follows:

where ω_x and ω_y are the angular velocity of the gyroscope around axes ox and oy respectively; J_x and J_y are the moments of inertia of a gyroscope around axes ox and oy; other torques represented in Table 1.

Equations (2) and (3) represent the system with two

variables that have a solution by Eq. (1). The practical solution and tests of Eqs. (1) – (3) are well matched that represents the validation of the correctness of the new analytical approach.

3. Conclusion

The gyroscope theory in classical mechanics was unsolvable for a long time. The known mathematical models for gyroscope theory contain many assumptions and simplifications of the unexplainable motions of gyroscopic devices. New studies demonstrate the torques generated by the centrifugal, common inertial and Coriolis forces in the spinning rotor play a critical role along with the change in the angular momentum. The action of each force is formulated by the mathematical model for the several problems and clearly described the physics of gyroscopic effects. The experimental tests of the motions and forces acting on the gyroscope match well the mathematical model. Thus, the new analytical approach represents the breakthrough gyroscope theory and covers the gap in classical mechanics.

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